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RUBBER SHEET GEOMETRY AND ITS APPLICATIONS

M. Vasuki* & A. Dinesh Kumar**

* Assistant Professor, Department of Mathematics, Srinivasan College of Arts and Science, Perambalur, Tamilnadu

** Assistant Professor, PG & Research Department of Mathematics, Khadir Mohideen College, Adirampattinam, Thanjavur, Tamilnadu

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Abstract:

General topology is important in many areas mainly in topological space which is used in data mining, computer-aided design, computational topology for geometric design and molecular design and quantum physics, etc… The concepts of somewhat b-continuous functions and somewhat b-open functions in topological spaces are introduced. Also given the relationships of these functions with other classes of function and given some examples. This paper deals with rubber sheet geometry and its applications.

Introduction:

"Topology" is an important branch of pure mathematics, sometimes referred to as "Rubber Sheet Geometry". The word topology comes from the Greek word 'Topo' means surface and 'Logy' means study (or) discover. Topology thus literally means the study of surface. The term "Topology" was introduced by Johann Benedict Listing in 1847. In 1930, James Waddell Alexander II and Hassler Whitney first expressed the idea that a surface is a topological space that is locally like a Euclidean plane. The area of topology dealing with abstract objects is referred to as General topology (or) point - set topology. General topology assumed its present form around 1940. It deals with the basic set – theoretic definitions used in topology. It is the foundation of most other branches of topology, including Algebraic topology, Geometric topology and Differential topology. Topological ideas are present in almost all areas of today's mathematics. One of the most basic structural concepts in topology is "Topological Space". It may be defined as a set of points, along with a set of neighbourhoods for each points. This paper is to discuss about the concept of b-functions in topological space and analyze some theorems and definitions based on topological spaces.

Preliminaries:

Definition:

A topology on a set X is a collection of τ of subset ofX having the following properties

- Φ and X are inτ
- The union of elements of any sub collection of τ is in τ
- The intersection of the elements of any finite sub collection of$\tau$is in τ .

A set X for which topologyτ has been specified is called Topological space.

Example:

Let X={a,b,c} then this set has 2^3 elements then τ ={{},{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c}}. To verify that τ is a topology on X or not.

Proof:

To prove that: τ is a topology on X or not.

Axiom:

Φ and X are inτ .

Axiom:

- $\Phi \cup \{a\}$= {a} $\in \tau$
- $\Phi \cup \{b\}$= {b} $\in \tau$
- $\Phi \cup$ {c} = {c}$\in \tau$
- $\Phi \cup X = X \in \tau$

Axiom:

- $\Phi \cap \{a\} = \{\Phi\} \in \tau$
- $\Phi \cap \{b\} = \{\Phi\} \in \tau$
- $\Phi \cap X = \{\Phi\} \in \tau$
- $\Phi \cap$ {a,b} = $\{\Phi\} \in \tau$

All the three axioms are satisfied.

Hence, $\tau$is a topology.

Definition:

If X is a topological space with topologyτ we say that a subset U of X is an**open set** of X if U $\in \tau$.

Note:

A topological space is a set X together with collection of subsets of X is called open set such that

- Φ and X are both open

- The arbitary union of open sets are open
- The intersection of open sets are open.

Definition:

A subset of a topological space X is said to be **closed** if the set X-A is open.

Definition:

Let X be a topological space and A be a subset of X. The interior of A is defined as the union of all open sets are contained in A. i.e) Int (A) = union of all open sets contained in A.

Example:

Let X = {a,b,c}
A = {a}
τ = {X,Φ,{a},{b},{a,b}}
ζ = {X,Φ,{b,c},{c,a},{c}}
Int (A) = Φ ∪ {a}
i.e) Int (A) = {a}

Definition:

Let X be a topological space and A be a subset of X. The **closure of A**is defined as the intersection of all closed sets containing A. i.e) cl(A) = Intersection of all closed sets containing A .

Example:

Let X = {a,b,c}
A = {a}
τ= {X,Φ,{a},{c},{a,c}}
ζ = {X,Φ,{b,c},{a,b},{b}}
cl (A) = X ∩ {a,b}
i.e) cl (A) = {a,b}

Definition:

If X is a set, **a basis** for the topology on X is a collection$\mathscr{B}$ of subsets of X (called basis elements) such that

- For each x ∈ X , there is atleast one basis element B containing x
  i.e) x ∈ X such that x ∈ B
- If x belongs to the intersection of two basis elements B_1 and B_2 , there is a basis element B_3 containing x such that $B_3 \subset B_1 \cap B_2$
  i.e) $x \in B_3 \subset B_1 \cap B_2$

Definition:

Let X be a topological space with topologyτ . If Y is a subset of X, the collection τ_y = {Y ∩ U / U ∈ τ}is a topology on Y called the subspace topology .

Definition:

If A is a subset of the topological space X and if x is a point of X, then x is a limit point of A if every neighbourhood of x intersects A in some point other than x itself. It is also called as "cluster point" or "point of accumulation"

Definition:

Let X and Y be topological spaces. A function f: X→Y is said to be continuous if for each open subset V of Y, the set f^{-1} (V) is an open subset of X.

Definition:

Let X and Y be topological spaces. Let f: X →Y be a bijection and the inverse function is a f^{-1} : Y→ X are continuous, then f is called a homeomorphism.

Example:

The function f: ℝ → ℝ given by f(x)=3x+1 is a homeomorphism. If wedefine g: ℝ → ℝ by the equation g (y) = $\frac{1}{3}$(y-1) .

Then, f (g(y)) = f ($\frac{1}{3}$(y-1)) = 3 ($\frac{1}{3}$(y-1)) +1= y – 1 + 1= y

g (f(x)) = g (3x+1)= $\frac{1}{3}$ (3x+1- 1) = x

Here, f (g(y)) = y and g (f(x)) = x , for all real numbers x and y .
Therefore, f is bijective and g = f^{-1}.

Definition:

A subset 'A' of a space X is said to be semi-open if A ⊆ cl (int (A)).

Example:

Let X = {a,b,c}
τ = {X,Φ,{a},{c},{a,c}}
ζ = {X, Φ,{b,c},{a,b},{b}}
Let A = X
Int (A) = X
cl (int (A)) = cl (X) = X
Therefore, A ⊆ cl (int (A))
⇒ X is semi-open.

Let X = Φ
Int (A) = Φ
cl (int(A)) = cl (Φ) = Φ
Therefore, A ⊆ cl (int (A))
⇒ Φis semi-open .
Let A = {a}
Int (A) = {a}
cl (int(A)) = cl ({a}) = {a,b}
Therefore, A ⊆ cl (int (A))
⇒ {a} is semi-open .
Let A = {b}
Int (A) = Φ
Cl (int (A)) = cl (Φ) = Φ
Therefore, A ⊆ cl (int (A))
⇒ {b} is not semi-open .
Let A = {c}
Int (A) = {c}
cl (int (A)) = cl ({c}) = {b,c}
Therefore, A ⊆ cl (int (A))
⇒ {c} is semi-open .
Let A = {a,b}
Int (A) = {a}
cl (int (A)) = cl ({a}) = {a,b}
Therefore, A ⊆ cl (int (A))
⇒ {a, b} is semi-ope .
Let A = {a,c}
Int (A) = {a,c}
cl (int (A)) = cl ({a, c}) = X
Therefore, A ⊆ cl (int (A))
⇒ {a, c} is semi-open .
Let A = {b,c}
Int (A) = {c}
cl (int (A)) = cl ({c}) = {b,c}
Therefore, A ⊆ cl (int (A))
⇒ {b, c} is semi-open .
Hence, the collection of semi-open sets
⇒{X,Φ,{a},{c},{a,b},{a,c},{b,c}} .

Definition:
The complement of a semi-open set is said to be semi-closed set.

Example:
Let X = {a,b,c}
τ = {X, Φ,{a},{c},{a,c}} ζ = {X, Φ,{b,c},{a,b},{b}}
The collection of semi-open sets of ⇒ X = {X,Φ,{a},{c},{a,b},{a,c},{b,c}}
The collection of semi-closed sets of ⇒ X = {X,Φ,{b,c},{a,b},{c},{b},{a}} .

Note:
Sint (A) = Semi interior of A = Union of all semi-open sets contained in A.

Example:
Let X = {a,b,c}
τ = {X,Φ,{a},{c},{a,c}} ζ= {X, Φ,{b,c},{a,b},{b}}
The collection of semi-open sets of ⇒ X = {X,Φ,{a},{c},{a,b},{a,c},{b,c}}
Let A = {a,b}
Sint (A) = Sint ({a,b}) = {a,b}

Note:
Scl (A) = Semi closure of A = Intersection of all semi-closed sets containing A .

Example:
Let X = {a,b,c}
τ= {X,Φ,{a},{c},{a,c}} ζ = {X, Φ,{b,c},{a,b},{b}}
The collection of semi-closed sets of ⇒ X = {X,Φ,{b,c},{a,b},{c},{b},{a}} .
Let A = {b}
Scl (A) = Scl ({b}) = {b}.

Result:
Every open set is semi-open

Proof:
Let A be open, then, A = int (A)
⇒ cl (A) = cl (int (A)) …………… (1)

Since, $A \subseteq cl\,(A)$ .... (2)

$A \subseteq cl\,(A) = cl\,(int\,(A))$ [from (1) and (2)]

$\Rightarrow$ $A \subseteq cl\,(int\,(A))$

$\Rightarrow$ A is semi-open

Hence, every open set is semi-open.

Remark:

A semi-open set is need not be an open set.

Example:

Let X = {a,b,c}

τ = {X,Φ,{a},{c},{a,c}} ζ = {X, Φ,{b,c},{a,b},{b}}

The collection of semi-open sets of ⇒ X = {X,Φ,{a},{c},{a,b},{a,c},{b,c}}

Since, {a,b} is semi-open but not open .

Definition:

There exist a continuous function defined on [0, 1] which maps the interval onto a two dimensional region in the plane. Such a function is called a peano-curve (or) space-filling curve.

Definition:

A topological space X is said to be a hausdroff space if for each pair x_1, x_2 of distinct points of X there exists neighbourhood u1 and u_2 of x_1 and x_2 respectively that are disjoint.

Definition:

A subset 'A' of a space X is said to be dense in X if $\overline{A} = X$.

Example:

Let X = {1, 2, 3, 4}

τ = {Φ, X, {1, 2}, {3, 4}}

Let A = {2, 3}

D (A) = {1, 4}

∴ Dense is $\overline{A} = X$

i.e) $\overline{A} = A \cup D\,(A) = \{2, 3\} \cup \{1, 4\} = \{1, 2, 3, 4\} = X$

Therefore, $\overline{A} = X$

Definition:

A metric on a set X is a function d: X x Y → R having the following properties

- $d\,(x,y) \geq 0$ for all $x,y \in X$; equality holds iff $x = y$
- $d\,(x,y) = d\,(y,x)$ for all $x , y \in X$
- $d\,(x,y) + d\,(y,z) \geq d\,(x,z)$ for all $x , y \in X$

[Triangle Inequality]

Definition:

A function f: X→Y is said to be somewhat semi continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \Phi$ there exist a semi open set V in X such that $V \neq \Phi$ and $V \subseteq f^{-1}(U)$.

Definition:

A function f : X→Y is said to be somewhat semi open function provided that for $U \in \tau$ and $U \neq \Phi$ there exists a semi open set V in Y such that $V \neq \Phi$ and $V \subseteq f(U)$.

Definition:

Let f: A → X x Y be given the equation $f(a) = (f_1(a), f_2(a))$. Then f is continuous iff the functions f_1 : A → X and f_2 : A → Y are continuous and the maps f_1 and f_2 are called the co-ordinate functions of f.

Definition:

If $\mathscr{B}$ is the collection of all open intervals in the real line (a, b) = {x / a < x < b} . Here the topology generated by $\mathscr{B}$ is called the standard topology on the real line. It is denoted by $\mathscr{R}$.

Definition:

A function f from the metric space (X, d_x) to the metric space (Y, d_y) is said to be uniformly continuous. If given $\varepsilon > 0$, there is a $\delta > 0$ such that for every pair of points x_0 , x_1 of X,

$$d_x(x_0,x_1) < \delta \Rightarrow d_y(f(x_0),f(x_1)) < \varepsilon.$$

Definition:

If A and B are two subsets of the topological space X and if there is a continuous function f : X → [0,1] such that f(A) = {0} and f(B) = {1} it is called as completely regular.

Definition:

Let $\mathscr{B}$ be the collection of all half open intervals of the form [a,b) = {x /a ≤ x < b} this topology is called the lower limit topology on $\mathscr{R}$ and it is denoted by $\mathscr{R}_1$.

Definition:

A connected space is a topological space that can be represented as the union of two (or) more disjoint non-empty open subsets.

Definition:

Let X and Y be topological spaces. The Product topology on X x Y is the topology having as basis collection $\mathcal{B}$ of all sets of the form U x V, where U is an open subset of X and V is an open subset of Y. It is also called as "Tychonoff topology".

Definition:

Let $\Pi_1 : X \times Y \to X$ be defined by the equation $\Pi_1 (X,Y) = x$ and let $\Pi_2 : X \times Y \to Y$ be defined by the equation $\Pi_2 (X,Y) = y$. The maps Π_1 and Π_2 are called the projections of X x Y onto its first and second factors respectively.

Somewhat b - Continuous Functions:

Definition:

Let (X,τ) and (Y,σ) be any two topological spaces .A function $f : X \to Y$ is said to be somewhat b-continuous function if for every $U \in \sigma$ and $f^{-1}(U) \neq \Phi$ there exists a b-open set V in X such that $V \neq \Phi$ and $V \subseteq f^{-1}(U)$.

Example:

Let X = {a,b,c}

τ = {X, Φ,{a},{c},{a,c}}

ζ = {X, Φ,{b,c},{a,b},{b}}

Let Y = {1,2,3}

σ = {Y, Φ,{1},{3},{1,3}}

ζ^1 = {Y, Φ,{2,3},{1,2},{2}}

The collection of b-open sets ⇒ {X, Φ,{a},{c},{a,c},{a,b},{b,c}}

Consider a function $f : (X, \tau) \to (Y, \sigma)$ be defined by ⇒ f (a) =1,f (b) = 2,f (c) =3 .

Let U ={1} ∈ σ and f^{-1} (U) = {a}≠ Φ

Then there exists a b-open set V ={a} ≠ Φ ∈ X such that $V \subseteq f^{-1}$ (U)

Let U ={3} ∈ σand f^{-1} (U) = {c} ≠ Φ .

Then there exists a b-open set V ={c} ≠ Φ ∈ X such that $V \subseteq f^{-1}$ (U)

Let U ={1,3} ∈ σ and f^{-1} (U) = {a,c} ≠ Φ .

Then there exists a b-open set V ={a,c} ≠ Φ ∈ X such that $V \subseteq f^{-1}$ (U) .

⇒ f is somewhat b-continuous function .

Hence, f is somewhat b-continuous function.

Hence the proof

Theorem:

Every somewhat semi-continuous function is somewhat b-continuous function.

Proof:

Consider a function f : X→Y

Let f be somewhat semi-continuous function

To Prove: f is somewhat b-continuous function

Let U be anyopen set in Y such that $f^{-1}(U) \neq \Phi$. Since f is somewhat semi-continuous function, there exists a semi-open set V in X such that $V \neq \Phi$and $V \subseteq f^{-1}(U)$

Since every semi-open set is b-open, there exists a b-open set V such that $V \neq \Phi$ and $V \subseteq f^{-1}(U)$

Thus for $U \in \sigma$ and $f^{-1}(U) \neq \Phi$ there exists a b-open set V in X such that $V \neq \Phi$ and $V \subseteq f^{-1}(U)$

⇒ f is somewhat b-continuous function. Hence proved.

Theorem:

Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y, \sigma) \to (Z,\eta)$ be any two functions. If f is somewhat b-continuous function and g is continuous function, then $g \circ f$ is somewhat b-continuous function.

Proof:

Let f: $(X,\tau) \to (Y,\sigma)$ be somewhat b-continuous function and g: $(Y, \sigma) \to (Z,\eta)$ be continuous function

To Prove: $g \circ f$ is somewhat b-continuous function

i.e) $g \circ f : (X,\tau) \to (Z,\eta)$ is somewhat b-continuous function

Let $U \in \eta$. suppose that $g^{-1}(U) \neq \Phi$,

Since $U \in \eta$ and g is continuous function $g^{-1}(U) \in \sigma$

Suppose that $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) \neq \Phi$

Then by hypothesis, f is somewhat b-continuous function , then there exists a b-openset V in X such that $V \neq \Phi$ and $V \subseteq f^{-1}(g^{-1}(U))$

But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$

i.e) $V \subseteq f^{-1}(g^{-1}(U))$

$V \subseteq (g \circ f)^{-1}(U) \Rightarrow V \subset (g \circ f)^{-1}(U)$

Therefore $g \circ f$ is somewhat b-continuous function. Hence proved.

Definition:

Let M be a subset of a topological space (X,τ) .Then M is said to be **b-dense** in X if there is no proper b-closed set C in X such that $M \subset C \subset X$.

Example:

Let X = {a,b,c}

τ = {X,Φ,{a},{c},{a,c}}

ζ = {X,Φ,{a,b},{b,c},{b}}

The collection of b-closed sets

⇒ {X,Φ,{b,c},{a,b},{c},{b},{a}}

Let M = {a}

There exist a proper b-closed sets C = {a,b} in X such that M ⊂C ⊂ X .
⇒ {a} is not b-dense in X .
Let M = {b}
There exist a proper b-closed sets C = {b,c} in X such that M ⊂C ⊂ X .
⇒ {b} is not b-dense in X .
Let M = {c}
There exist a proper b-closed sets C = {c} in X such that M ⊂C ⊂ X .
⇒ {c} is not b-dense in X .
Let M = {a,b}
There exist a proper b-closed sets C = {a,b} in X such that M ⊂C ⊂ X .
⇒ {a,b} is not b-dense in X .
Let M = {b,c}
There exist a proper b-closed sets C = {b,c} in X such that M ⊂C ⊂ X .
⇒ {b,c} is not b-dense in X .
Let M = {a,c}
There exist a no proper b-closed sets C in X such that M ⊂C ⊂ X
⇒ {a,c} is b-dense in X .
∴ The collection of b-open sets ⇒ {X,Φ,{a,c}} .

Theorem:

Let f: $(X,\tau) \to (Y,\sigma)$ be a function .Then the following are

- f is somewhat b-continuous function
- If C is a closed subset of Y such that $f^{-1}(C) \neq X$,then there is a proper b-closed subset D of X such that
 $D \supset f^{-1}(C)$.
- If M is a b-dense subset of X then f (M) is a dense subset of Y.

Proof:

To Prove: 1) → 2)
Assume that f is somewhat b-continuous
To Prove: If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a properb-closed subset D of X such that $D \supset f^{-1}(C)$

Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then Y – C is an open set in Y such that

$$f^{-1}(Y - C) = f^{-1}(Y) - f^{-1}(C) = X - f^{-1}(C) \neq \Phi$$

By hypothesis, Since f is somewhat b-continuous then there exists a b-open set V in X such that $V \neq \Phi$ and $V \subseteq f^{-1}(Y - C) = X - f^{-1}(C)$ ⇒ $X - V \supset f^{-1}(C)$
[∵ Since V is open in X ⇒ X – V is closed in X]
Let X – V = D is a b-closed set in X ⇒ $D \supset f^{-1}(C)$.
There is a proper b-closed subset D of X such that $D \supset f^{-1}(C)$
This proves 2).
To Prove: 2) → 3)
Let M be a b-dense set in X
To prove that: f (M) is dense in Y
Suppose we assume that f (M) is not dense in Y then there exists a proper closed set C in Y such that f (M) ⊂ C ⊂ Y

$$M \subset f^{-1}(C) \subset f^{-1}(Y) = X \Rightarrow f^{-1}(C) \neq X$$

By 2), there exist a proper b-closed set D of X such that $f^{-1}(C) \subset D$
⇒ $M \subset f^{-1}(C) \subset D \subset X$ ⇒ $M \subset f^{-1}(C) \subset X$ ⇒ M is not b-dense in X
Which is a contradiction. Therefore, M is b-dense in X ⇒ f (M) is dense in Y .
To Prove: 3) → 2)
Suppose that 2) is not true , this means there exists a closed set C in Y such that $f^{-1}(C) \neq X$, then there is no proper b-closed set D in X such that $f^{-1}(C) \subseteq D$
i.e) there is no proper b-closed set D in X such that
$f^{-1}(C) \subset D \subset X$.⇒ $f^{-1}(C)$ is b-dense in X
By 3), $f(f^{-1}(C)) = C$ is dense in Y
Which is a contradiction, because C is closed in Y. Hence 2) is true.
To Prove: 2) → 1)
i.e) To prove: f is somewhat b-continuous
Let $U \in \sigma$ and $f^{-1}(U) \neq \Phi$, then Y – U is closed in Y .

$$f^{-1}(Y - U) = f^{-1}(Y) - f^{-1}(U) = X - f^{-1}(U) \neq \Phi$$

By hypothesis of 2), there exists a proper b-closed set D such that $D \supset f^{-1}(Y - U)$

$$\Rightarrow D \supset X - f^{-1}(U) \Rightarrow X - D \subset f^{-1}(U)$$

Since D is b-closed set

$$\Rightarrow X - D \text{ is b-open.}$$

There exists a b-open X – D in X such that $X - D \subset f^{-1}(U)$
∴ f is somewhat b-continuous function. Hence, 1) proves. Hence Proved.

Theorem:

Let (X,τ) and (Y,σ) be any two topological spaces X = A ∪ B where A and B are open subsets of X and f: (X,τ) → (Y, σ) be a function such that f/A and f/B are somewhat b-continuous functions, then f is somewhat b-continuous function.

Proof:

To Prove: f is somewhat b-continuous function

Let U be any open set in (Y, σ) such that f^{-1} (U) ≠ Φ then $(f/A)^{-1}$ (U) ≠ Φ(or)$(f/B)^{-1}$ (U) ≠ Φ (or) both $(f/A)^{-1}$ (U) ≠ Φ and $(f/B)^{-1}$ (U) ≠ Φ.

Case (i): Suppose $(f/A)^{-1}$ (U) ≠ Φ

Since f/A is somewhat b-continuous. Therefore there exist a b-open set V in A such that

V ≠ Φ and V ⊂ $(f/A)^{-1}$ (U)

i.e) V ⊆ f^{-1} (U)

Since V is b-open in A and A is open in X. ⇒ V is b-open in X.

Therefore, there exist a b-open set V in X such that V ≠ Φ and V ⊂ f^{-1} (U).

Thus, f is somewhat b-continuous function.

Case (ii): Suppose $(f/B)^{-1}$ (U) ≠ Φ

Since f/B is somewhat b-continuous

Therefore there exist a b-open set V in B such that V ≠ Φ and V ⊂ $(f/B)^{-1}$ (U)

i.e) V ⊆ f^{-1} (U)

Since V is b-open in B and B is open in X ⇒V is b-open in X.

Therefore, there exist a b-open set V in X such that V ≠ Φ andV ⊂ f^{-1} (U). Thus, f is somewhat b-continuous function.

Case (iii): Suppose $(f/A)^{-1}$ (U) ≠ Φ and $(f/B)^{-1}$ (U) ≠ Φ

From case (i) and case (ii), ⇒ f is somewhat b-continuous function

Hence by above 3 cases ⇒ f is somewhat b-continuous function.

Definition:

A topological space X is said to be b-separable if there exists a countable subset B of X which is b-dense in X.

Theorem:

If f is somewhat b-continuous function from X onto Y and if X is b-separable then Y is separable.

Proof:

Let f: X → Y be somewhat b-continuous function

Since X is b-separable

Then by definition of b-separable, "There exists a countable subset B of X which is b-dense in X"

Since f: X → Y be somewhat b-continuous function and B is a b-dense subset of X

Then f(B) is dense in Y, Since B is countable and the image of a countable set is countable ⇒ f (B) is countable

⇒ f (B) is countable which is dense in Y .

Therefore, Y is separable. Hence f is somewhat b-continuous function from X onto Y and if X is b-separable then Y is separable . Hence proved.

Somewhat b - Open Functions:

Definition:

A function f : (X, τ) → (Y, σ) is said to be somewhat b-open function provided that for U ∈ τ and U ≠ Φ , there exists a b-open set V in Y such that V ≠ Φ and V ⊆ f (U) .

Example:

Let X = {a,b,c}

τ = {X, Φ,{a},{c},{a,c}}

ζ = {X, Φ,{b,c},{a,b},{b}}

Let Y = {1,2,3}

σ = {Y, Φ,{1},{3},{1,3}}

ζ^1 = {Y, Φ,{2,3},{1,2},{2}}

We know that, The collection of b-open sets

⇒ {X, Φ,{a},{c},{a,c},{a,b},{b,c}}

The collection of b-open sets in

⇒ Y = {Y,Φ,{1},{3},{1,3},{1,2},{2,3}}

A function f : (X, τ) → (Y, σ) be defined by

f (a) = 1, f (b) = 2, f (c) = 3.

Let U = {a} ∈ τ and U ≠ Φ

V = {1} ∈ Y

Then, V = {1} = f (a) = f (U)

⇒ V ⊆ f (U)

Let U = {c} ∈ τ and U ≠ Φ

V = {3} ∈ Y

Then, V = {3} = f (c) = f (U)

⇒ V ⊆ f (U)

Let U = {a,c} ∈ τ and U ≠ Φ
V = {1,3} ∈ Y
Then, V = {1,3} = f (a,c) = f (U)
⇒ V ⊆ f (U)
∴ f is somewhat b-open function.

Theorem:

Every somewhat semi-open function is somewhat b-open function.

Proof:

Let f : (X, τ) → (Y, σ) be a somewhat semi-open function
To prove: f is somewhat b-open function
Let U ∈ τ and U ≠ Φ
Since f is somewhat semi-open function, there exists a semi-open set V in Y such that V ≠ Φ and V ⊂ f (U). Since every semi-open set is b-open. Therefore , there exists a b-open set V in Y such that V ≠ Φ and V ⊂ f (U) ⇒ f is somewhat b-open function. Hence proved .

Theorem:

If f : (X, τ) → (Y, σ) is an open map and g : (Y, σ) → (Z, η) is somewhat b-open map then g o f : (X, τ) → (Z, η) is somewhat b-open map

Proof:

Let f : (X, τ) → (Y, σ) is an open map and g : (Y, σ) → (Z, η) is somewhat b-open map
To Prove: g o f : (X, τ) → (Z, η) is somewhat b-open map
Let U ∈ τ , suppose thatU ≠ Φ, Since f is an open map, f (U) is open and f (U) ≠ Φ. Thus, f (U) ∈ σ and f (U) ≠ Φ. Since, g is somewhat b-open map and f (U) ∈ σ such that f (U) ≠ Φ, there exists a b-open set V ∈ η such that V ⊂ g (f (U)) ⇒ V ⊂ (g o f) (U)
Hence, g o f is somewhat b-open function. Hence proved.

Theorem:

If f : (X, τ) → (Y, σ) is a one-one and onto mapping, then the following conditions are equivalent

- f is somewhat b-open map
- If C is a closed subset of X such that f (C) ≠ Y, then there is a b-closed subset D of Y such that D ≠ Y and D ⊃ f (U).

Proof:

Let f : (X, τ) → (Y, σ) is a one-one and onto mapping
To prove: i) → ii)
Let C be any closed subset of X such that f (C) ≠ Y
Then, X-C is open in X and X-C ≠ Φ.
Since fis somewhat b-open, then there exists a b-open set
V ≠ Φin Y such that V ⊂ f (X-C) ⇒ V ⊂ f (U) – f (C) ⇒ V ⊂ Y – f (C) ⇒ Y – V ⊃ f (C)
Put D = Y – V, since V is b-open in Y ⇒ Y – V is b-closed in Y ⇒ D is b-closed in Y
We claim that, D ≠ Y
Suppose D = Y ⇒ D = Y – V. Then, V ≠ Φ
Which is a contradiction

∴ D ≠ Y

Thus if C is a closed subset of X such that f (C) ≠ Y, there is a b-closed subset D of Y such that D ≠ Y and D ⊃ f (C)
To prove: ii) → i)
Let U be an non-empty open set in X such that U ≠ Φ
Put C = X – C, Then C is a closed subset of X

f (X – U) = f (X) – f (U) = Y – f (U) = f (C) ⇒ f (C) ≠ Φ

By ii), there is a b–closed subset D of Y such that
D ≠ Y and f (C) ⊂ D
Put V = Y – D
Since D is b-closed in Y ⇒ Y – D is b-open in Y
V is b-open in Y and V ≠ Φ
Since f (C) ⊂ D ⇒ Y – D ⊂ Y – f (C) ⇒ V ⊂ f (X) – f (C) ⊂ f (X-C) ⊂ f (U)

⇒V ⊂ f (U)

Thus for U ∈ τ and U ≠ Φ, there is a b-open set V in Y and V ≠ Φ such that V ⊂ f (U) .

⇒ f is somewhat b-open map. Hence Proved.

Theorem:

Let (X, τ) and (Y, σ) be any two topological space and X = A ∪ B where A and B are open subsets of X and f : (X, τ) → (Y, σ) be a function such that f /A and f /B are somewhat b-open, then f is also somewhat b-open function.

Proof:

To prove: f : (X, τ) → (Y, σ) is somewhat b-open function
Let U be any open subset of (X, τ) such that U ≠ Φ. Since X = A ∪ B, either U ∩ A ≠ Φ (or) U ∩ B ≠ Φ(or) both U ∩ A ≠ Φand U ∩ B ≠ Φ

Since U is open in (X, τ) ⇒U is open in both $(A, \tau / A)$ and $(B, \tau / B)$
Case (i): Suppose that $U \cap A \neq \Phi$ where $U \cap A$ is open in τ / A
Since f /A is somewhat b-open function, there exists a b-open set $V \in (Y, \sigma)$ such that $V \subset f(U \cap A) \subset f(U)$ ⇒ f is somewhat b-open function.
Case (ii): Suppose that $U \cap B \neq \Phi$ where $U \cap B$ is open in τ / B
Since f /B is somewhat b-open function, there exists a b-open set $V \in (Y, \sigma)$ such that $V \subset f(U \cap B) \subset f(U)$ ⇒ f is somewhat b-open function.
Case (iii): Suppose that both $U \cap A \neq \Phi$ and $U \cap B \neq \Phi$
Then obviously, f is somewhat b-open function. From the case i) and case ii),
Thus, f is somewhat b-open function. Hence Proved.

Conclusion:

In this paper, the concepts of Somewhat b-continuous functions and Somewhat b-open functions in topological spaces are introduced. Also given the relationships of these functions with other classes of function and given some examples. General topology is important in many areas mainly in topological space which is used in data mining, computer-aided design, computational topology for geometric design and molecular design and quantum physics, etc… Therefore, all functions defined in this projects will have many possibilities of applications by using topological spaces. Thus open, closed, semi-open, semi-closed sets are introduced and related examples and theorems are also studied.

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